Int J Theor Phys (2011) 50: 164–170
DOI 10.1007/s10773-010-0504-1

Einstein-Rosen Universe with Wet Dark Fluid in General Relativity

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Received: 21 July 2010 / Accepted: 17 September 2010 / Published online: 6 October 2010

Abstract Field equations in the presence of wet dark fluid distribution are obtained in general relativity with the aid of Einstein-Rosen cylindrically symmetric metric.

A static vacuum model and non-static stiff fluid model are investigated. The physical and geometrical properties of the stiff fluid model are also studied.

Keywords General relativity · Wet dark fluid · Stiff fluid

1 Introduction

Type Ia supernovae observational data suggest that the universe is dominated by two dark components containing dark matter (DM) and dark energy (DE). Dark matter, a matter without pressure, is mainly used to explain galactic curves and large-scale structure formation, while dark energy, an exotic energy with negative pressure, is used to explain the present cosmic accelerating expansion. To understand the origin of dark matter & dark energy and its nature is one of the greatest astronomical/cosmological problems of the 21st century. The nature of both components remains unknown, and in the near future we can hope that the Large Hadron Collider (LHC) will be able to provide hints on the nature of DM & DE.

The understanding of cosmology has been revolutionized by recently observed astronomical phenomena. Consequences of combined analysis of Ia Supernovae (SNeIa) observations [12, 29–31, 33, 37, 38, 40, 41, 51], galaxy cluster measurements [5] and cosmic microwave background (CMB) data [50] have shown that the expansion of our present universe is accelerating rather than slowing down. This late time cosmic acceleration cannot be explained by the four known fundamental interactions in the standard models. Within the framework of Einstein's general relativity, an exotic component with negative pressure called as dark energy is invoked to explain these observed phenomena. The simplest dark energy candidate is the vacuum energy which is mathematically equivalent to cosmological

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constant Λ which has the equation of state $\omega = -1$. As per [11] "fine-tuning" and the "cosmic coincidence" are the two well known difficulties of the cosmological constant problems. There are different alternative theories for the dynamical dark energy scenario which have been proposed by scientists to interpret the accelerating universe. The dynamical scalar field models of dark energy includes quintessence [34, 57], k-essence based on earlier work of k-inflation [3, 9], phantom (ghost) field [8, 27] and quintom field [2, 14], tachyon field [1, 28, 42], dilatonic ghost condensate [17, 32]. The interacting dark energy models includes Chaplygin gas [6, 23], Holographic dark energy models [10, 16, 22, 25, 53, 59, 60] and Braneworld models [13, 39]. Freeman and Turner [15], Wang and Tegmark [54] established firmly that universe is actually undergoing an acceleration, with repulsive gravity of some strange energy-form i.e. dark energy at work. Dark energy, a "mysterious substance" pressure of which is "negative" and accounts for nearly 70% of total matter-energy budget of universe, but has no clear explanation. Setare [45–48], Setare and Saridakis [43, 44] have developed the idea of holographic dark energy. Recently the original agegraphic dark energy (OADE) and new agegraphic dark energy (NADE) models were proposed by Cai [7] and Wei and Cai [55, 56]. Karami *et al.* [24] introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. Gupta and Pradhan [19] proposed new candidate known as cosmological nuclear-energy as a possible suspect (candidate) for the dark energy. Mukhopadhyay *et al.* [26] studied higher dimensional dark energy investigation with variable Λ and G. However, so far, the nature of dark energy is still unclear. The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW model by Holman and Naidu [21]. The Bianchi type-I universe filled with dark energy from a wet dark fluid has been studied by Singh and Chaubey [49].

In this paper, we have studied cylindrically symmetric Einstein Rosen universe with wet dark fluid in general relativity. Some physical and kinematical properties of the model are also discussed.

2 Wet Dark Fluid

Holman and Naidu [21] studied the homogeneous, isotropic FRW case by using the wet dark fluid (WDF) as dark energy (DE) candidate. This model was in the spirit of the Generalized Chaplygin Gas (GCG) [18], where a physically motivated equation of state was offered with properties relevant for the dark energy problem. This was stemmed from an empirical equation of state proposed by Tait [52] and Hayward [20] to treat water and aqueous solution.

The equation of state for WDF is given in simple form as

$$p_{\text{WDF}} = \gamma(\rho_{\text{WDF}} - \rho^*) \tag{1}$$

It is motivated by the fact that it is a good approximation for many fluids, including water, in which initial attraction of the molecules makes negative pressure possible.

The parameters γ and ρ^* are taken to be positive and we restrict ourselves to $0 \le \gamma \le 1$. Note that if c_s denotes the adiabatic sound speed in WDF, then $\gamma = c_s^2$ [4].

To find the WDF energy density, we use the energy conservation equation

$$\rho^*_{\text{WDF}} + 3H(p_{\text{WDF}} + \rho_{\text{WDF}}) = 0 \tag{2}$$

Using the equation of state (1) and relation $3H = \frac{\dot{v}}{v}$ in above equation (2), we get

$$\rho_{\text{WDF}} = \frac{\gamma}{1+\gamma}\rho^* + \frac{D}{v(1+v)}, \tag{3}$$

where D is the constant of integration and v is the volume expansion.

WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a piece that red shifts as a standard fluid with an equation of state $p = \gamma\rho$.

We can show that if we take $D > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$. Thus, we get

$$\begin{aligned} p_{\mathrm{WDF}} + \rho_{\mathrm{WDF}} &= (1+\gamma)\rho_{\mathrm{WDF}} - \gamma\rho^* \\ &= (1+\gamma)\frac{D}{v^{(1+\gamma)}} \geq 0 \end{aligned} \tag{4}$$

Holman and Naidu [21] observed that their model is consistent with the most recent SNela data, the WMAP results as well as the constraints coming from measurements of the power spectrum. Here, they have considered both, the case where the dark fluid is smooth (i.e. only the CDM component clusters gravitationally) as well as the case where the dark fluid also clusters.

3 Metric and Solutions of Field Equations

We consider the non-static cylindrically symmetric Einstein-Rosen metric as

$$ds^2 = e^{2\alpha-2\beta}(dt^2 - dr^2) - r^2 e^{-2\beta} d\phi^2 - e^{2\beta} dz^2 \tag{5}$$

where α and β are both the functions of r and t only.

We denote the coordinates r, ϕ, z, t as x^1, x^2, x^3, x^4 respectively.

The Einstein field equations in general relatively are

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G T_{ij} \tag{6}$$

where R_{ij} is the Ricci tensor, R the scalar curvature.

Here we consider geometrized units so that $8\pi G = C = 1$.

The energy-momentum tensor of the source [21, 49] is given by

$$T_i^j = (\rho_{\mathrm{WDF}} + p_{\mathrm{WDF}}) u_i u^j - p_{\mathrm{WDF}} \delta_i^j, \tag{7}$$

where u^i is the flow vector satisfying

$$g_{ij} u^i u^j = 1 \tag{8}$$

In a co-moving system of coordinates, from (7) we find

$$T_1^1 = T_2^2 = T_3^3 = -p_{\mathrm{WDF}} \quad \text{and} \quad T_4^4 = \rho_{\mathrm{WDF}} \tag{9}$$

With the help of (7), (8) and (9), the field equations (6) for the metric (5) yield

$$\left[\beta_1^2 + \beta_4^2 - \frac{\alpha_1}{r}\right] e^{-2\alpha+2\beta} = -p_{\mathrm{WDF}} \tag{10}$$

$$[\alpha_{44} - \alpha_{11} - \beta_1^2 + \beta_4^2] e^{-2\alpha+2\beta} = -p_{\mathrm{WDF}} \tag{11}$$

$$\left[2\beta_{11} - 2\beta_{44} + 2\frac{\beta_1}{r} - \alpha_{11} + \alpha_{44} - \beta_1^2 + \beta_4^2\right] e^{-2\alpha+2\beta} = -p_{\mathrm{WDF}} \tag{12}$$

$$\left[\beta_1^2 + \beta_4^2 - \frac{\alpha_1}{r}\right] e^{-2\alpha+2\beta} = -\rho_{\mathrm{WDF}} \tag{13}$$

and

$$2\beta_1\beta_4 - \frac{\alpha_4}{r} = 0 \tag{14}$$

Equations (10) and (13) immediately imply that

$$p_{\mathrm{WDF}} = \rho_{\mathrm{WDF}} \tag{15}$$

Which shows that the solutions of the above field equations are possible only for stiff fluid (Zel'dovich fluid). As the above field equations are highly nonlinear in nature, therefore, it is difficult to obtain the explicit solution of the field equations for this case.

We, therefore, consider only two physically interesting cases as follows (namely):

(i) Static vacuum ($p_{\mathrm{WDF}} = \rho_{\mathrm{WDF}} = 0$) and
(ii) Non-static stiff fluid ($p_{\mathrm{WDF}} = \rho_{\mathrm{WDF}}$)

3.1 Static Vacuum Model

In this case, we have $p_{\mathrm{WDF}} = 0$, $\rho_{\mathrm{WDF}} = 0$ and α, β are function r only.

Therefore, in this case, the field equations (10)–(14) reduce to the following equations

$$\beta_1^2 - \frac{\alpha_1}{r} = 0 \tag{16}$$

$$\alpha_{11} + \beta_1^2 = 0 \tag{17}$$

$$2\beta_{11} + 2\frac{\beta_1}{r} - \alpha_{11} - \beta_1^2 = 0. \tag{18}$$

This admits the exact solution given by

$$\alpha = c_1 \log r + c_2, \qquad \beta = c_3 \log r + c_4 \tag{19}$$

After a suitable choice of co-ordinates and constants, Einstein-Rosen cylindrically symmetric metric (5) can be written as

$$ds^2 = r^{2(A-B)}(dt^2 - dr^2) - r^{2(1-B)} d\phi^2 - r^{2B} dz^2 \tag{20}$$

3.2 Non-static Stiff Fluid Model (Zel'dovich universe)

Stiff fluid (Zel'dovich fluid) can be regarded as a prefect fluid having energy-momentum tensor given by (7) characterized by (15) i.e. $p_{\mathrm{WDF}} = \rho_{\mathrm{WDF}}$.

The astrophysical importance of this equation of state is that it describes several important cases: e.g. radiation, relativistic degenerate Fermi gas and possible very dense matter [58].

Here we consider $p_{\mathrm{WDF}} = \rho_{\mathrm{WDF}}$ and α, β are functions of t only.

In this case the field equations (10)–(14) reduce to

$$(\beta_4^2) e^{-2\alpha+2\beta} = -p_{\mathrm{WDF}} \tag{21}$$

$$[\alpha_{44} + \beta_4^2] e^{-2\alpha+2\beta} = -p_{WDf} \tag{22}$$

$$[-2\beta_{44} + \alpha_{44} + \beta_4^2] e^{-2\alpha+2\beta} = -p_{WDf} \tag{23}$$

$$(\beta_4^2) e^{-2\alpha+2\beta} = -\rho_{\mathrm{WDF}} \tag{24}$$

and

$$\frac{\alpha_4}{r} = 0. \tag{25}$$

Which admit the exact solution given by

$$\alpha = a_1, \qquad \beta = a_2 t + a_3 \tag{26}$$

where a_1, a_2, a_3 are constants of integration.

After a suitable choice of co-ordinates and constant, cylindrically symmetric Einstein-Rosen metric (5) becomes

$$ds^2 = e^{-2T}(dt^2 - dr^2 - r^2 d\phi^2) - e^{2T} dz^2. \tag{27}$$

This represents a non-static Einstein-Rosen-Zel'dovich universe with wet-dark fluid in general relativity. It may be noted that the universe (27) has no initial singularity.

4 Physical and Geometrical Properties

The physical and kinematical quantities for the model (27) have the following expression:

The pressure and density in the model (27) is obtained as (with suitable choice of constants) $p = \rho = e^{2T}$

Spatial volume: $v = (-g)^{\frac{1}{2}} = re^{-2T}$

Expansion scalar: $\theta = u^i_{;i} = -e^T$

Shear scalar: $\sigma^2 = \frac{13}{6} e^{2T}$

Deceleration parameter: $q = -3\theta^{-2}\left[\theta_{;\alpha} u^\alpha + \frac{1}{3}\theta^2\right] = 2$

The model (27) has no finite singularity.

As $T \to \infty$, the spatial volume and the energy density tends to zero and infinity respectively.

The model (27) is a contracting model since the scalar expansion is always negative. Here $(\frac{\sigma^2}{\theta^2})$ does not vanish for a large value of T which implies that the model (27) anisotropic and does not approach isotropy.

The deceleration parameter q acts as an indicator of the existence of inflation of the model. If $q > 0$, the model decelerates in the standard way while $q < 0$ indicates inflation. Here positive value of q implies that the model (27) decelerates in a standard way.

To study the non-singular behavior of the model (27), we consider the Riemann-curvature invariant R for the metric (5) given by

$$R = 2e^{-2\alpha + 2\beta}\left[-\alpha_{11} + \alpha_{44} + \beta_{11} - \beta_{44} + \beta_4^2 - \beta_1^2 + \frac{\beta_1}{r}\right]$$

For the model (27) this is found to be

$$R = e^{2T}$$

Here $R \to \infty$ as $T \to \infty$. Hence, the model (27) is not singular in the finite past and in the infinite future.

5 Conclusion

We have studied the behavior of cylindrically symmetric Einstein-Rosen metric with wet dark fluid in general relativity. We have obtained Einstein-Rosen cylindrically symmetric static vacuum model with a wet dark fluid. Also a non-static non-singular led contracting cosmological model is obtained which decelerates in the standard way. It is important to note that the results obtained in this paper are identical with the results obtained by Reddy *et al.* [35, 36].

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